## (2) Confidence interval

The estimated proportion plus or minus its margin of error is called a confidence interval for the true proportion. The $95 \%$ confidence for a proportion is given by:
sample proportion - margin of error $\leq$ true proportion $\leq$ sample proportion + margin of error

$$
\begin{aligned}
& \hat{p}-z \sigma_{\hat{p}} \leq p \leq \hat{p}+z \sigma_{\hat{p}} \\
& \hat{p}-1.96 \sigma_{\hat{p}} \leq p \leq \hat{p}+1.96 \sigma_{\hat{p}} \\
& \hat{p}-1.96 \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \leq p \leq \hat{p}+1.96 \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}
\end{aligned}
$$

Where $n$ is the sample size, $p$ is the population proportion and $\hat{p}$ is the sample proportion.

We can state with $95 \%$ confidence that the true population, $p$, lies inside this interval. What this means is that if the same population was surveyed on numerous occasions and the confidence interval was calculated, then about $95 \%$ of these confidence intervals would contain the true proportion and about $5 \%$ of these confidence intervals would not contain the true proportion.

The end points of the $95 \%$ confidence are given by $\hat{p} \pm 1.96 \sigma_{\hat{p}}$ :


It is worth noting that when $p$ (or $\hat{p}$ instead of $p$ if $p$ is unknown) is close to $\frac{1}{2}$, a good approximation to the margin of error, at the $95 \%$ confidence level, is given by $E=z \sigma_{p}=\frac{1}{\sqrt{n}}$.

